

## Motion Analysis Homework

1.

A particle moves along the x-axis so that at time  $t$  its position is given by

$$x(t) = \sin(\pi t^2) \text{ for } -1 \leq t \leq 1.$$

- Find the velocity at time  $t$ .
- Find the acceleration at time  $t$ .
- For what values of  $t$  does the particle change direction?
- Find all values of  $t$  for which the particle is moving to the left.

2.

Suppose that the function  $f$  has a continuous second derivative for all  $x$ , and that  $f(0) = 2$ ,  $f'(0) = -3$ , and  $f''(0) = 0$ . Let  $g$  be a function whose derivative is given by  $g'(x) = e^{-2x}(3f(x) + 2f'(x))$  for all  $x$ .

- Write an equation of the line tangent to the graph of  $f$  at the point where  $x = 0$ .
- Is there sufficient information to determine whether or not the graph of  $f$  has a point of inflection when  $x = 0$ ? Explain your answer.
- Given that  $g(0) = 4$ , write an equation of the line tangent to the graph of  $g$  at the point where  $x = 0$ .
- Show that  $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$ . Does  $g$  have a local maximum at  $x = 0$ ? Justify your answer.

3.

Let  $h$  be a function defined for all  $x \neq 0$  such that  $h(4) = -3$  and the derivative of  $h$  is given by  $h'(x) = \frac{x^2 - 2}{x}$  for all  $x \neq 0$ .

- Find all values of  $x$  for which the graph of  $h$  has a horizontal tangent, and determine whether  $h$  has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
- On what intervals, if any, is the graph of  $h$  concave up? Justify your answer.
- Write an equation for the line tangent to the graph of  $h$  at  $x = 4$ .
- Does the line tangent to the graph of  $h$  at  $x = 4$  lie above or below the graph of  $h$  for  $x > 4$ ? Why?